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POLYNOMIAL COMPENSATION, INVERSION, AND APPROXIMATION OF
DISCRETE TIME LINEAR SYSTEMS

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Abstract

The least-squares transformation of a discrete-time multivariable linear system into a desired one by convolving the first with a polynomial system yields optimal polynomial solutions to the problems of system compensation, inversion and approximation. The polynomial coefficients are obtained from the solution to a so-called normal linear matrix equation, whose coefficients are shown to be the weighting patterns of certain linear systems. These, in turn, can be used in the recursive solution of the normal equation.

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1. Introduction

The transformation of a given linear system into one having desired properties, by cascading it with another linear system, has been known in control system design as cascade compensation (see, e.g., [1]). The problems of system inversion and approximation can also be formulated as system transformation problems by properly defining the roles of the systems involved. An exact causal transformation from the given to the desired system may not exist or may not be implementable because of complexity constraints and an approximate transformation of a relatively low complexity may be desired. However, finding an approximation within the class of rational linear systems of a given order is, typically, a difficult parameter optimization problem involving local extrema (see, e.g., [2]).

In this paper we consider the transformation of discrete time, multivariable linear systems by convolving them with systems having finite weighting patterns or, equivalently, polynomial transfer matrices. In contrast to the continuous-time case, discrete time polynomial systems do not present a realization problem. Such systems have been used extensively in statistics and in signal processing for predicting and filtering discrete time processes (see, e.g., [3], [4]). The proposed approach may be viewed as the compensator analog of Levinson's (polynomial) approximation to Wiener's (rational) filter [5]. The least-squares approximation criterion yields a globally optimal solution to the system transformation problem at hand in the form of the solution to a so-called normal linear matrix equation, similar to the one arising in the filtering problem.

Representing the given and the desired systems in state-space, we obtain the coefficients of the normal equation and of the approximation error in closed explicit forms and show that they are the weighting patterns of certain

linear systems. These systems can, in turn, be used in mechanizing the recursive solution of the polynomial coefficients by a multivariable version of the Levinson procedure [6]. The applicability of the polynomial transformation method to the problems of system compensation, inversion and approximation is discussed and illustrated by numerical examples.

2. Least-Squares Polynomial Compensation, Inversion and Approximation of Linear Systems

System Compensation

Given two discrete-time linear systems, having rational transfer matrices of the same dimensions $\Omega(z)$ and $\Pi(z)$, it is desired to find a system having a polynomial transfer matrix

$$\Phi(z) = \sum_{i=0}^n T_i z^{-i} \quad (2.1)$$

that, cascaded with $\Omega(z)$ will approximate $\Pi(z)$ in some sense. Denoting by $\{p_k\}$, $\{q_k\}$ and $\{t_k\}$, $k \geq 0$, the weighting patterns corresponding, respectively, to $\Omega(z)$, $\Pi(z)$ and $\Phi(z)$, an approximation measure is defined by the L_2 norm

$$\begin{aligned} \|\Omega(z)\Phi(z) - \Pi(z)\|_2 &\equiv \left\{ \text{tr} \int_0^{2\pi} [\Omega(e^{j\omega})\Phi(e^{j\omega}) - \Pi(e^{j\omega})]^\dagger \right. \\ &\quad \times [\Omega(e^{j\omega})\Phi(e^{j\omega}) - \Pi(e^{j\omega})] d\omega \Big\}^{1/2} \\ &= \left\{ \text{tr} \sum_{k=0}^{\infty} (p_k * t_k - q_k)^T (p_k * t_k - q_k) \right\}^{1/2} \end{aligned} \quad (2.2)$$

where tr denotes trace, $(\cdot)^*$ denotes complex conjugate transpose, $(\cdot)^T$ denotes real transpose, and $*$ denotes the convolution operation, that is

$$p_k * t_k = \sum_{i=0}^k p_{k-i} t_i$$

Denoting

$$\underline{t} = [T_0^T \ T_1^T \ \dots \ T_n^T]^T$$

it can be verified that the approximation measure (2.2) takes the form

$$\|\Omega(z) - \Pi(z)\|_2 = \{\text{tr}(\underline{L}\underline{t} - \underline{q})(\underline{L}\underline{t} - \underline{q})^T\}^{1/2} \quad (2.3)$$

where

$$L = \begin{bmatrix} p_0 & & & 0 \\ p_1 & p_0 & & \\ \vdots & & \ddots & \\ p_n & p_{n-1} & \dots & p_0 \\ p_{n+1} & p_n & \dots & p_1 \\ \vdots & & & \end{bmatrix} \quad \text{and} \quad \underline{q} = \begin{bmatrix} q_0 \\ q_1 \\ \vdots \end{bmatrix}$$

Minimizing (2.3) with respect to \underline{t} yields the normal equation

$$L^T L \underline{t} = L^T \underline{q} \quad (2.4)$$

Denoting

$$\underline{s} = L^T \underline{q} = \begin{bmatrix} s_0 \\ s_{-1} \\ \vdots \\ s_{-n} \end{bmatrix} \quad (2.5)$$

and

$$R = L^T L = \begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_n \\ r_1^T & r_0 & r_1 & \cdots & r_{n-1} \\ r_2^T & r_1^T & r_0 & \cdots & r_{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_n^T & r_{n-1}^T & r_{n-2}^T & \cdots & r_0 \end{bmatrix} \quad (2.6)$$

where

$$r_i = \sum_{k=0}^{\infty} p_{k+i}^T p_k \quad \text{and} \quad s_{-i} = \sum_{k=0}^{\infty} p_k^T q_{k+i} \quad (2.7)$$

the normal equation becomes

$$R \underline{t} = \underline{s} \quad (2.8)$$

The optimal approximation error is obtained by substituting (2.4) into (2.3) as

$$\varepsilon = \left[\text{tr} \left(v_0 - \sum_{i=0}^n s_{-i}^T t_i \right) \right]^{1/2} \quad (2.9)$$

where t_1 is defined by the solution of (2.8) and

$$v_0 = \sum_{k=0}^{\infty} q_k^T q_k \quad (2.10)$$

Suppose that both $\Omega(z)$ and $\Pi(z)$ are stable, then

$$\text{tr} \sum_{k=0}^{\infty} p_k^T p_k < \infty \quad \text{and} \quad \text{tr} \sum_{k=0}^{\infty} q_k^T q_k < \infty$$

implying that both r_1 and s_1 have finite limit values and, consequently, the normal equation (2.8) and the optimal error (2.9) are well defined. We note that as the polynomial degree n is increased to infinity, the optimal error vanishes, provided that an exact causal linear transformation between $\Omega(z)$ and $\Pi(z)$ exists. This can be seen immediately from the fact that the error corresponding to the optimal sequence $\{t_k\}$ is minimal, while the one corresponding to the impulse response of the exact transformation converges to zero. Suppose that for some $l \geq 0$ we have $p_k = 0$, $0 \leq k \leq l - 1$ and $p_l \neq 0$ and for some $m \geq 0$, $q_k = 0$, $0 \leq k \leq m - 1$ and $q_m \neq 0$. Then the systems $\Omega(z)$ and $\Pi(z)$ are said to have, respectively, l and m delays or "zeros at infinity" (see, e.g., [7], p. 449). If $l > m$, an exact causal rational transformation from $\Omega(z)$ to $\Pi(z)$ clearly does not exist and the error corresponding to a polynomial compensator will not vanish even when its degree is increased to infinity. If, on the other hand, $l < m$, the compensation problem can be simplified somewhat by replacing $\Omega(z)$ by $z^{-(m-l)}\Omega(z)$ and then multiplying the resulting compensator by $z^{-(m-l)}$. This is illustrated by an example in section 4.

System Inversion.

The problem of system inversion is a special case of the compensation problem. Suppose that it is desired to find a polynomial system of the form (2.1) that approximates the inverse of a given rational system $\Omega(z)$. An inversion criterion is defined by minimizing the norm

$$\|\Omega(z)\Phi(z) - I\|_2 = \left\{ \text{tr} \sum_{k=0}^{\infty} (p_k * t_k - I\delta_k)^T (p_k * t_k - I\delta_k) \right\}^{1/2} \quad (2.11)$$

which yields the normal equation

$$R\underline{t} = \underline{b} \quad (2.12)$$

where R is defined by (2.6) and \underline{b} is the $n + 1$ block matrix

$$\underline{b} = [p_0 \ 0 \ \dots \ 0]^T$$

The optimal inversion error is obtained from (2.9) as

$$\epsilon = \{\text{tr}(I - p_0^T t_0)\}^{1/2} \quad (2.13)$$

where t_0 is obtained from the solution of (2.12). In analogy with the equivalent problem of solving an overdetermined linear matrix equation using the least-squares criterion, the term pseudo-inverse seems appropriate for the resulting approximate inverse system. If for some $l \geq 0$ we have $p_k = 0$, $0 \leq k \leq l - 1$ and $p_l \neq 0$, the solution of (2.12) is trivial, $\underline{t} = 0$. However, if the purpose of the inverse system is to reconstruct the input sequence from the output sequence, a more sensible approach to the inversion

problem is to invert a shifted version of the system, having no delays. This means that $\Omega(z)$ is replaced by $z^l \Omega(z)$ and the sequence $\{p_k\}$ is replaced in equations (2.11)-(2.13) by the sequence $\{p_k^{(l)}\}$, $k \geq 0$, where $p_k^{(l)} = p_{k+l}$. The resulting approximate inverse system should then be viewed as operating on a record of the output sequence, advanced l time units (u_{k+l} replacing u_k) to produce the input sequence.

System Approximation

Suppose that it is desired to approximate a given rational system $\Omega(z)$ by a polynomial system $\Phi(z)$. Writing the approximation measure as

$$\|\Phi(z) - \Omega(z)\|_2 = \left\{ \text{tr} \sum_{k=0}^{\infty} t_k * \delta_k - p_k \right\}^{1/2}$$

the problem can be seen to be a special case of the polynomial transformation problem with I and $\Omega(z)$ replacing $\Omega(z)$ and $\Pi(z)$, respectively. The resulting normal equation (2.8) has

$$R = I \quad \text{and} \quad \underline{s} = [p_0^T \ p_1^T \ \dots \ p_n^T]^T$$

yielding

$$\underline{t} = [p_0^T \ p_1^T \ \dots \ p_n^T]^T \quad (2.14)$$

It follows that the best polynomial approximation of a linear system has the first markov parameters of the system as polynomial coefficients.

When the given system has a weighting pattern of long duration (that is, of relatively large values for a relatively long time) a polynomial

approximation, which has a finite weighting pattern, will require many polynomial coefficients for a sensible representation. An alternative approach that does not enforce a finite weighting pattern but still maintains the mathematical simplicity of the polynomial transformation, is to use the inversion criterion as a system approximation criterion. This means that the approximate system is the one whose inverse, when convolved with the given system, approximates the identity matrix. The solution of the least squares polynomial inversion problem yields an inverse system $\Phi(z)$ of the form (2.1). The question of interest is whether $\Phi(z)$ has a causal inverse; that is, whether there exists a proper rational matrix $H(z)$ that satisfies

$$H(z)\Phi(z) = I \quad (2.15)$$

It follows from a fundamental result for polynomial matrix equations (see, e.g., [7], p. 387) that a rational matrix $H(z)$ that satisfies (2.15) exists if and only if T_0 has full column rank and that one such matrix is given by

$$H(z) = \Theta^{-1}(z) \quad (2.16)$$

where $\Theta(z)$ consists of the elements of $\Phi(z)$ corresponding to a non-singular minor of T_0 having a maximal dimension. Since $\Theta(z)$ is a polynomial matrix, its inverse is obviously proper. We note that in the single input single output case the approximate system (2.16) is of the autoregressive (all pole) type.

3. The Normal Equation Coefficients and its Recursive Solution

Let (A, B, C, D) and (F, G, H, E) be state-space realizations of $\Omega(z)$ and $\Pi(z)$, respectively. Then, noting that

$$p_k = \begin{cases} D & k = 0 \\ CA^{k-1}B & k \geq 1 \end{cases} \quad \text{and} \quad q_k = \begin{cases} E & k = 0 \\ HF^{k-1}G & k \geq 1 \end{cases}$$

it can be verified that

$$r_i = \begin{cases} D^T D + B^T P B & i = 0 \\ B^T (A^T)^{i-1} C^T D + B^T (A^T)^i P B & i \geq 1 \end{cases} \quad (3.1)$$

and

$$s_i = \begin{cases} D^T E + B^T Q G & i = 0 \\ D^T H F^{i-1} G + B^T Q F^i G & i \geq 1 \end{cases} \quad (3.2)$$

where

$$P = \sum_{k=0}^{\infty} (A^T)^k C^T C A^k \quad (3.3)$$

and

$$Q = \sum_{k=0}^{\infty} (A^T)^k C^T H F^k \quad (3.4)$$

the term v_0 in the optimal error, defined by (2.10) can be written as

$$v_0 = E^T E + G^T V G \quad (3.5)$$

where

$$V = \sum_{k=0}^{\infty} (F^T)^k H^T H F^k \quad (3.6)$$

It can be seen that the matrices P , Q and V , defined by equations (3.3), (3.4) and (3.5) are central to the normal equation and the optimal error. Suppose that both A and F have their eigenvalues strictly inside the unit circle. Then it can be verified that the matrices P , Q and V are the limit values of the matrices P_m , Q_m and V_m that satisfy the recursive equations

$$P_{m+1} = A^T P_m A + C^T C \quad (3.7)$$

$$Q_{m+1} = A^T Q_m F + C^T H \quad (3.8)$$

$$V_{m+1} = F^T V_m F + H^T H \quad (3.9)$$

yielding the algebraic equations

$$P = A^T P A + C^T C \quad (3.10)$$

$$Q = A^T Q F + C^T H \quad (3.11)$$

and

$$V = F^T V F + H^T H \quad (3.12)$$

Equations (3.10) and (3.12) are Lyapunov equations that can be solved by standard methods. Equation (3.11) is not a Lyapunov equation, as the matrices involved are generally not symmetric. It can be solved by equating its two sides term by term, which results in a linear equation of the form $LX = z$, where L has dimension $n^2 \times n^2$. The three equations (3.10)-(3.12) can be put in the form of a single Lyapunov equation

$$\begin{bmatrix} P & Q \\ Q^T & V \end{bmatrix} = \begin{bmatrix} A^T & 0 \\ 0 & F^T \end{bmatrix} \begin{bmatrix} P & Q \\ Q^T & V \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} + \begin{bmatrix} C^T C & C^T H \\ H^T C & H^T H \end{bmatrix} \quad (3.13)$$

and solved by standard procedures. These equations can also be solved by propagating equations (3.7)-(3.9) until convergence is observed. Other alternatives for the solution of Lyapunov equations can be found in the literature (see, e.g., [8], p. 67). We note that, as can be seen from (3.3), (3.4) and (3.6), P has full rank if and only if (A, C) is observable, V has full rank if and only if (F, H) is observable, and Q has full rank if and only if both are observable.

It can be seen from (3.1) and (3.2) that the coefficient sequence $\{r_i\}$, $i \geq 0$ is the weighting pattern of the system

$$\begin{aligned} x_{k+1} &= A^T x_k + (C^T D + A^T P B) u_k \\ y_k &= B^T x_k + (D^T D + B^T P B) u_k \end{aligned} \quad (3.14)$$

and, similarly, the coefficient sequence $\{s_i\}$, $i \geq 0$ is the weighting pattern of the system $(F, G, D^T H + B^T Q F, D^T E + B^T Q G)$.

The symmetric Toeplitz structure of the normal equation (2.8) can be exploited in its solution. Employing a multivariable version of the Levinson algorithm (see, e.g., [6], pp. 241-244) the solution for a polynomial of degree n can be obtained from the solution for a polynomial of degree $n-1$. Such procedure can be used for calculating a polynomial of a given degree without resorting to matrix inversion, or for recursively increasing the polynomial order, until an error criterion is met. We note that the key step of the Levinson procedure is an operation of the type

$$a_n = \sum_{i=0}^n r_{n+1-i} t_{n,i} \quad (3.15)$$

where $t_{n,i}$ is the i 'th parameter of a polynomial of order n and r_{n+1-i} is defined by (3.1). In the filtering application of the Levinson algorithm, both the memory requirement of the $\{r_k\}$ sequence and the execution of (3.15) make the algorithm impractical for large n values. On the other hand, for the present application, both problems can be circumvented by noting that a_n is the final output of the system (3.14) when the input is the sequence $\{t_{n,0}, t_{n,1}, \dots, t_{n,n}, 0\}$. It follows that (3.15) can be executed for each n by passing the polynomial coefficients of the preceding iteration through the system (3.14).

4. Examples

The numerical implementation of the proposed solutions to the compensation, inversion and approximation problems has been examined by solving numerical examples. It should be emphasized that, in general, the polynomial degree needed for a satisfactory accuracy depends on the specific systems involved.

Example 1: System Compensation

Suppose that it is desired to compensate a system whose transfer function is given by

$$\Omega(z) = \frac{(z + 0.2)(z - 0.5 \pm 0.2j)(z + 0.02 \pm 0.1j)}{(z - 0.2)(z + 0.1)(z + 0.6 \pm 0.3j)(z - 0.1 \pm 0.3j)}$$

by a polynomial compensator, so as to approximate the desired system

$$\Pi(z) = \frac{1}{(z + 0.3521)(z - 0.8521)}$$

The impulse responses of both systems are shown in Figure 1. State space representations for $\Omega(z)$ and $\Pi(z)$ have been obtained in controller canonical form and the calculations described in sections 2 and 3 were carried out using standard software. We note that since $\Pi(z)$ has two zeros at infinity, it was multiplied by z^2 and the resulting compensator was multiplied by z^{-2} , in order to reduce the number of optimized polynomial coefficients. The resulting polynomial compensator coefficients for degrees 4, 9, 14 and 19 are listed below.

$$\underline{t}_4 = [0 \quad 0 \quad 0.8082 \quad 1.6574 \quad 1.8709 \quad 1.3566 \quad 0.5323]^T$$

$$\underline{t}_9 = [0 \quad 0 \quad 1.0015 \quad 2.3625 \quad 3.3676 \quad 3.8086 \quad 3.7758 \quad 3.3627 \quad 2.7198$$

$$1.9271 \quad 1.0806 \quad 0.3568]^T$$

$$\underline{t}_{14} = [0 \quad 0 \quad 1.0001 \quad 2.3603 \quad 3.3723 \quad 3.8409 \quad 3.8777 \quad 3.6051 \quad 3.2113$$

$$2.7782 \quad 2.3621 \quad 1.9735 \quad 1.6050 \quad 1.2353 \quad 0.8560 \quad 0.4754 \quad 0.1565]^T$$

and

$$\underline{t}_{19} = [0 \quad 0 \quad 1.0000 \quad 2.3600 \quad 3.3716 \quad 3.8394 \quad 3.8751 \quad 3.6015 \quad 3.2078$$

$$2.7785 \quad 2.3753 \quad 2.0179 \quad 1.7116 \quad 1.4520 \quad 1.2317 \quad 1.0417 \quad 0.8719$$

$$0.7120 \quad 0.5501 \quad 0.3824 \quad 0.2128 \quad 0.0701]^T$$

The approximation errors for the four compensators were obtained from (2.9) as

$$\epsilon_4 = 1.1985, \quad \epsilon_9 = 0.5726, \quad \epsilon_{14} = 0.2544 \quad \text{and} \quad \epsilon_{19} = 0.1141$$

The impulse responses of the compensated system are shown in Figure 2, along with the impulse response of the desired system. It can be seen that the compensation quality improves as the compensator's degree is increased and that the compensator emphasizes matching the large initial response values, as should be expected.

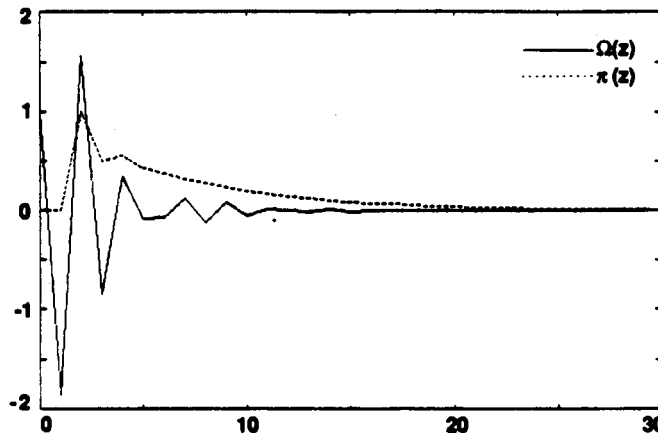


Figure 1 The impulse responses of the given system $\Omega(z)$ and the desired system $\Pi(z)$.

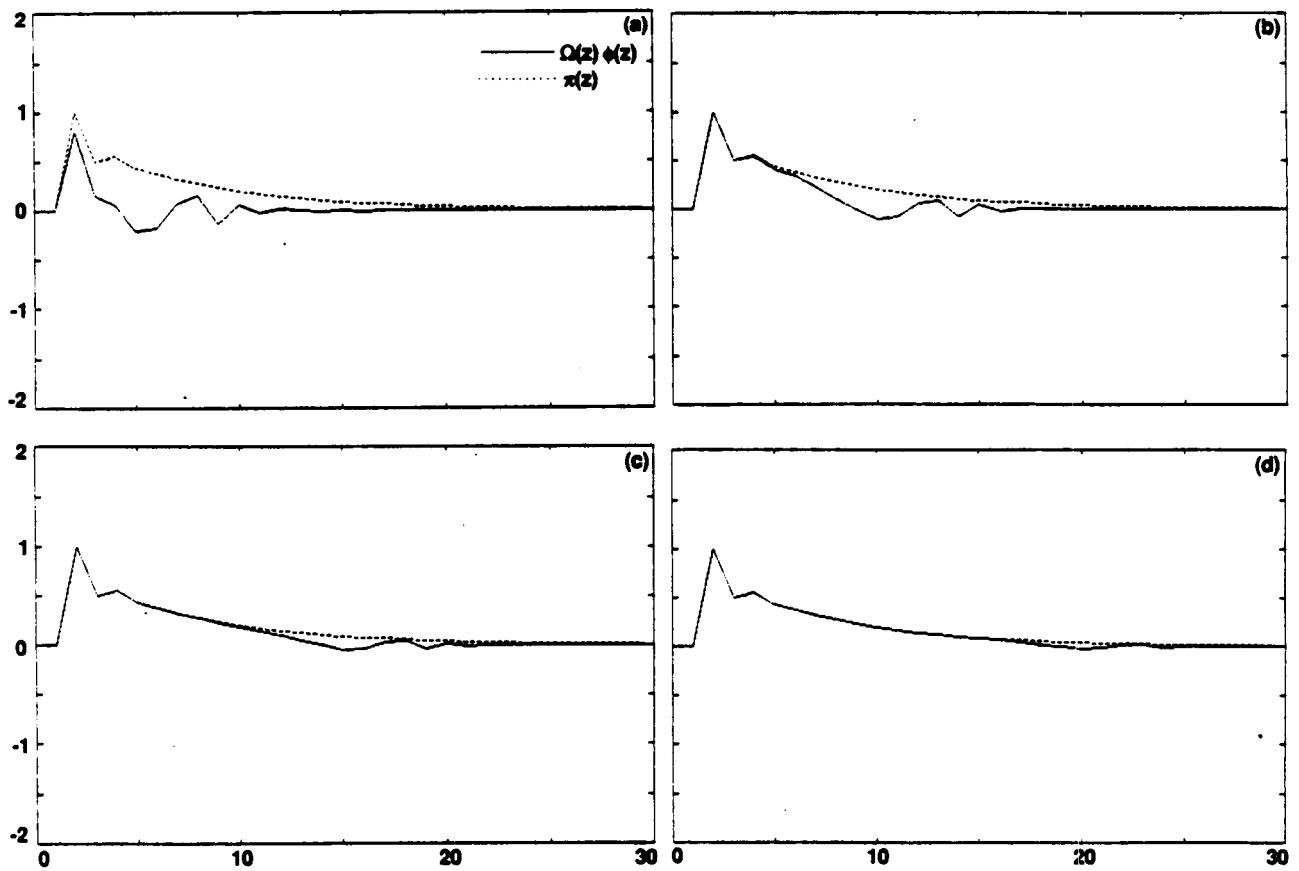


Figure 2 The impulse responses of the compensated system $\Omega(z)\Phi(z)$ for polynomial compensators of degrees (a) 4, (b) 9, (c) 14, and (d) 19.

Example 2: System Inversion and Approximation

Inverse polynomials of degrees 3 and 6 for the system $\Omega(z)$ of example 1 were derived using the proposed method. These are given by

$$\Phi_3(z) = 0.8493 + 1.3887z^{-1} + 1.0356z^{-2} + 0.3555z^{-3}$$

and

$$\begin{aligned}\Phi_6(z) = & 0.9970 + 1.8479z^{-1} + 1.8620z^{-2} + 1.3874z^{-3} + 0.8499z^{-4} + 0.3834z^{-5} \\ & + 0.1067z^{-6}\end{aligned}$$

The inversion errors associated with these and with polynomials of degrees 7 and 9 were obtained by (2.13) as

$$\epsilon_3 = 0.3888 \quad \epsilon_6 = 0.0548 \quad \epsilon_7 = 0.0200 \quad \epsilon_9 = 0.0011$$

The impulse responses of $\Phi_3^{-1}(z)$ and $\Phi_6^{-1}(z)$ are shown in Figure 3 along with the impulse response of $\Omega(z)$. It can be seen that a close approximation is obtained even for these relatively low degree polynomials. A graphically perfect match was obtained for polynomials of degrees 7 or higher. In contrast to this approximation by inversion, approximation by (2.14) can be seen to be inadequate.

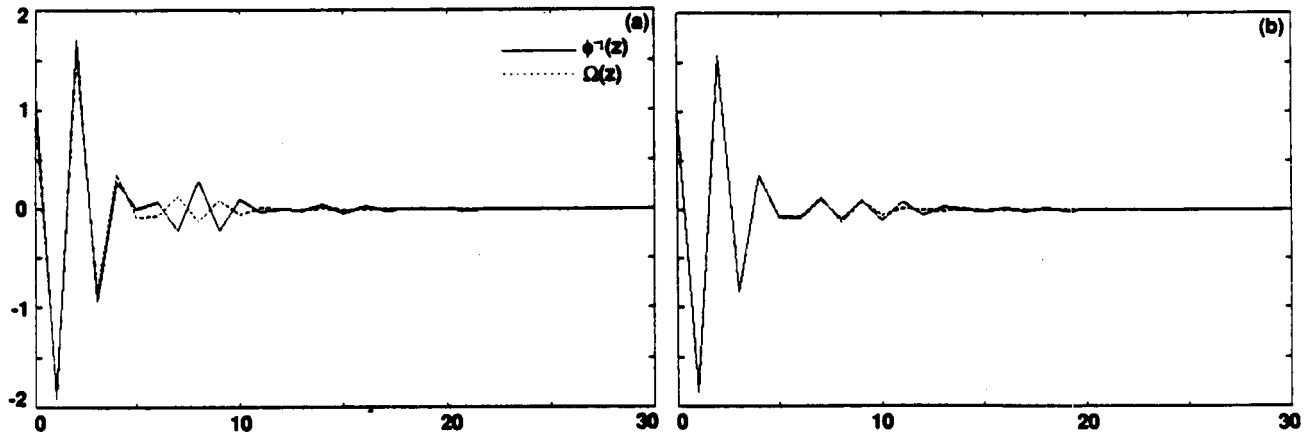


Figure 3. The impulse responses of the inverse approximate $\phi^{-1}(z)$ of $\Omega(z)$ for polynomial degrees (a) 3 and (b) 6.

5. Conclusion

The least-squares parameters of a polynomial system that approximately transforms a discrete-time linear system into another satisfy a linear equation, whose coefficients have been derived in terms of the state space parameters of the systems involved. The resulting method has been shown to apply to the problems of system compensation, inversion and approximation.

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